7 Reference datums, embeddings, and object reference models

7.1 Introduction

This International Standard specifies reference datums as geometric primitives in position-space that are used to model aspects of object-space through a process termed reference datum binding. A reference datum binding is an identification of a reference datum in position-space with a corresponding constructed entity in object-space (see 7.2). Reference datums for celestial bodies of interest are specified in Annex D.

A normal embedding is a distance-preserving function from position-space to object-space. A normal embedding establishes a position-space model of object-space. The image of a bound reference datum under a normal embedding may or may not coincide with the constructed entity of the reference datum binding. If they coincide, the reference datum binding and the normal embedding are said to be compatible (see 7.3).

A set of bound reference datums can be selected so as to be compatible with only one normal embedding. In this way, a set of bound reference datums with properly constrained relationships can specify a unique normal embedding. Such a constrained set of bound reference datums is called an object reference model. Object reference models that use the same set of reference datum primitives and similar binding constraints are abstracted in the notion of an object reference model template. Object reference model templates provide a uniform method of object reference model specification (see 7.4).

Object reference models for celestial objects of interest are specified in Annex E. For these celestial objects, one object reference model is designated as the reference model for the object. The transformation from each object reference model to the reference model for the object is termed the reference transformation. A reference transformation is a type of similarity transformation. Similarity transformation templates are defined in 7.3.3 to facilitate the specification of reference transformations. Time-independent reference transformations for celestial object reference models are also specified in Annex E.

Object-specific rules to bind reference datums in a way that is compatible with the binding constraints of an object reference model template are defined in 7.5. These object-specific binding rules are used to provide a uniform method of specifying object reference models for specific dynamically-related celestial bodies.

7.2 Reference datums

7.2.1 Introduction

A *reference datum* (RD) is a geometric primitive in position-space that is used to model an aspect of object-space through a process termed RD binding. In this International Standard, the reference datum concept is defined for 1D, 2D, and 3D position-spaces. In the 2D and 3D cases, this International Standard specifies a small set of reference datums for use in its own specifications. This set is not intended to be exhaustive. Users of this International Standard may specify additional reference datums by registration in accordance with Clause 13.

7.2.2 Reference datums

In this International Standard, an RD geometric primitive is expressed in terms of analytic geometry in position-space. RDs are designed to correspond to constructed entities of similar geometric type in an object-space through a process called RD binding (see 7.2.5). These geometric types are limited to a point, a directed curve, or an oriented surface. The analytic form of the position-space representation and its corresponding object-space geometric representation are described by category and position-space dimension in Table 7.1. An RD of a given category is specified by the parameters and/or the analytic expression of its position-space representation.

Table 7.1 — RD categories

RD category	Position-space representation			Object-space representation
	1D	2D	3D	
Point	(a) real a	(a, b) real a, b	(a, b, c) real a, b, c	a point in the object-space
Directed curve		$\boldsymbol{p} = \boldsymbol{F}(t)$, $\boldsymbol{F}$ is smooth and $\mathbf{R}^2$ valued. Direction at $\boldsymbol{p_0} = \boldsymbol{F}(t_0)$ is $\boldsymbol{n} = \frac{d\boldsymbol{F}}{dt}(t_0)$.	$\boldsymbol{p} = \boldsymbol{F}(t)$, $\boldsymbol{F}$ is smooth and $\mathbf{R}^3$ valued. Direction at $\boldsymbol{p_0} = \boldsymbol{F}(t_0)$ is $\boldsymbol{n} = \frac{d\boldsymbol{F}}{dt}(t_0)$.	a curve in the object-space with a designation of direction along the curve
Oriented surface			Implicit definition $f(\boldsymbol{p}) = 0$. Positive side of surface (orientation): $f(\boldsymbol{p}) > 0$	a surface in the object-space with a designation of one side as positive

This International Standard specifies 2D and 3D RDs by RD category in Table 7.4 through Table 7.8. The specification elements of those tables are defined in Table 7.2. 3D RDs based on ellipsoids are described in 7.2.3 and 7.2.4 and specified in Annex D with specification elements defined in Table 7.9. Table 7.3 is a directory of RD specification tables or, in the case of 3D RDs based on ellipsoids, RD specification directories.

Table 7.2 — RD specification elements

Element	Definition
RD label	The label for the RD (see 13.2.2).
RD code	The code for the RD (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Description	A description of the RD including any common name for the concept.
Position-space representation	The analytic formulation of the RD in position-space.

Table 7.3 — RD specification directory

Position-space dimension	RD category	Table number
2D	point	Table 7.4
3D	point	Table 7.5
2D	directed curve	Table 7.6
3D	directed curve	Table 7.7
3D	oriented surface	Table 7.8 and Table 7.10

Table 7.4 — 2D RDs of category point

RD label	RD code	Description	Position-space representation
ORIGIN_2D	1	Origin in 2D	(0,0)
X_UNIT_POINT_2D	2	x-axis unit point in 2D	(1,0)
Y_UNIT_POINT_2D	3	y-axis unit point in 2D	(0,1)

Table 7.5 — 3D RDs of category point

RD label	RD code	Description	Position-space representation
ORIGIN_3D	4	Origin in 3D	(0,0,0)
X_UNIT_POINT_3D	5	x-axis unit point in 3D	(1,0,0)
Y_UNIT_POINT_3D	6	y-axis unit point in 3D	(0,1,0)
Z_UNIT_POINT_3D	7	z-axis unit point in 3D	(0,0,1)

Table 7.6 — 2D RDs of category directed curve

RD label	RD code	Description	Position-space representation
X_AXIS_2D	8	x-axis in 2D	$\boldsymbol{F}(t) \equiv (t,0)$
Y_AXIS_2D	9	y-axis in 2D	$\boldsymbol{F}(t) \equiv (0,t)$

Table 7.7 — 3D RDs of category directed curve

RD label	RD code	Description	Position-space representation
X_AXIS_3D	10	x-axis in 3D	$\boldsymbol{F}(t) \equiv (t,0,0)$
Y_AXIS_3D	11	y-axis in 3D	$\boldsymbol{F}(t) \equiv (0,t,0)$
Z_AXIS_3D	12	z-axis in 3D	$\boldsymbol{F}(t) \equiv (0,0,t)$

Table 7.8 — 3D RDs of category oriented surface

RD label	RD code	Description	Position-space representation
XY_PLANE_3D	13	xy-plane	$f(x,y,z) \equiv z = 0$
XZ_PLANE_3D	14	xz-plane	$f(x,y,z) \equiv y = 0$
YZ_PLANE_3D	15	yz-plane	$f(x,y,z) \equiv x = 0$

7.2.3 Ellipsoidal RDs

The RDs specified in this International Standard include RDs based on oblate ellipsoids, prolate ellipsoids, and tri-axial ellipsoids. These RDs are 3D and of category oriented surface. These RDs are specified based upon certain geometrically-defined parameters. The position-space representations of oblate and prolate ellipsoid RDs are expressed in the form:

$$f(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1 = 0. \tag{7.1}$$

When $a \geq b$, an RD of this form is an *oblate ellipsoid RD* with major semi-axis a and minor semi-axis b as illustrated in Figure 7.1.

Spheres shall be considered as a special case of oblate ellipsoids. If $a = b$, an oblate ellipsoid RD may be called a *sphere RD*. In this case, the value $r = a = b$ is the radius of the sphere RD.

NOTE In general usage, spheres are a limiting case of oblate, prolate, and tri-axial ellipsoids. To remove ambiguity, in this International Standard spheres are a special case of oblate ellipsoids only.

When $a < b$, an RD of this form is a *prolate ellipsoid RD* with major semi-axis b and minor semi-axis a, as illustrated in Figure 7.1.

Instead of specifying the parameters of an oblate ellipsoid RD as the major semi-axis a and the minor semi-axis b, it is both equivalent and sometimes convenient to use the major semi-axis a and the flattening f as defined in Equation (7.2). The minor semi-axis b may be expressed in terms of the major semi-axis a and the flattening f as in Equation (7.3). The flattening of a sphere RD is zero ($f = 0$).

flattening definition: $$f \equiv \frac{a-b}{a} \tag{7.2}$$

minor semi-axis relationship: $$b = a - a\,f \tag{7.3}$$

The position-space representation of a tri-axial ellipsoid RD is expressed in the form:

$$f(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0. \tag{7.4}$$

The semi-axes a, b, and c shall be positive non-zero and $a \neq b \neq c \neq a$.

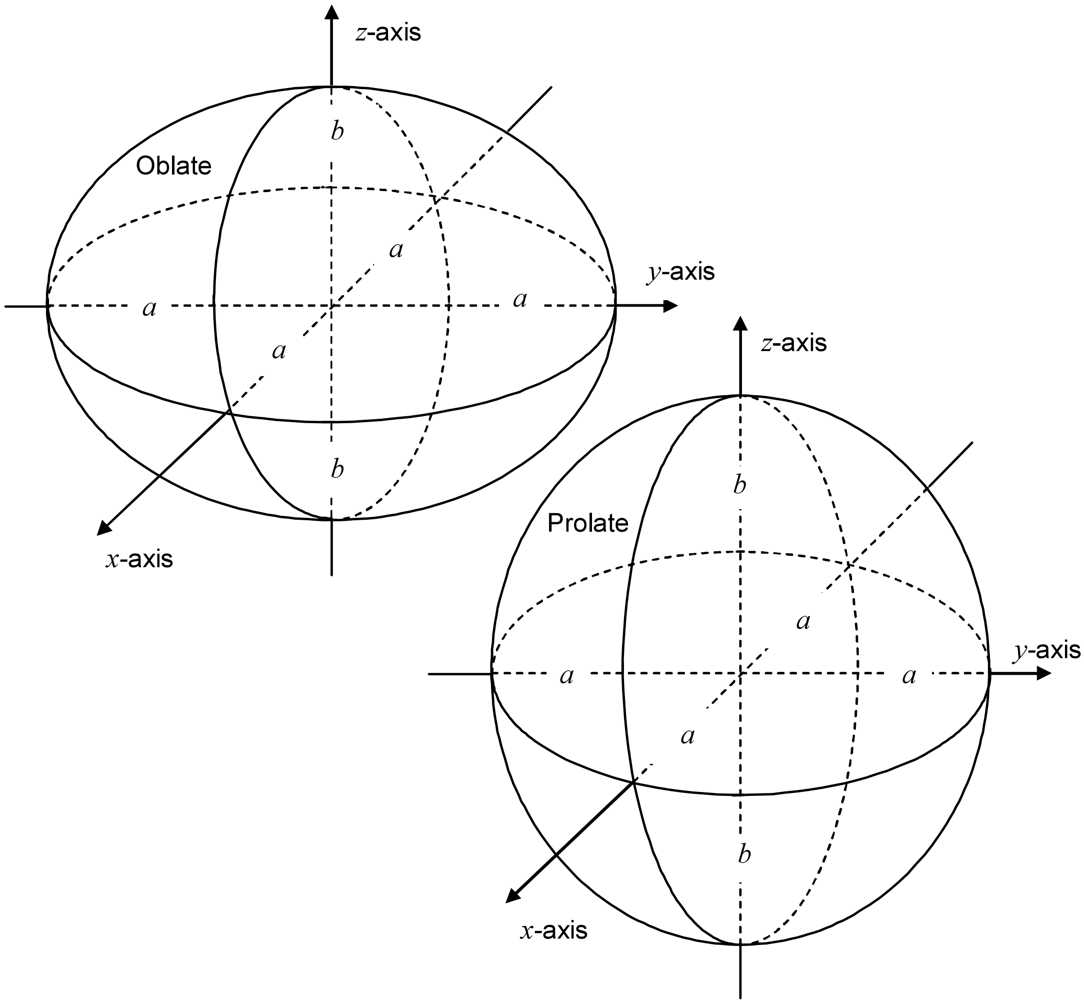

Figure 7.1 — Oblate and prolate ellipsoids

7.2.4 RDs associated with physical objects

In the case of ellipsoid RDs intended for modelling physical objects of interest, published parameter values for these RDs are used. The specification of these RDs includes the published ellipsoid parameters and the identification of the associated physical object. The specification elements for physical object RDs are defined in Table 7.9.

Table 7.9 — Physical object RD specification elements

Element	Specification
RD label	The label for the RD (see 13.2.2).
RD code	The code for the RD (see 13.2.3).
Description	The description including the name as published or as commonly known.
Physical object	The name of the physical object.

Element	Specification	
Parameters	Oblate ellipsoid case (including the sphere case)	Major semi-axis, a Flattening, f
	Prolate ellipsoid case	Minor semi-axis, a Major semi-axis, b
	Tri-axial ellipsoid case	x-semi-axis, a y-semi-axis, b z-semi-axis, c
	RD parameters shall be specified by value or by reference (see 13.2.5). If by value, the value(s) shall be followed by an error estimate expressed in one of the following forms: a) error estimate: unknown b) error estimate: assumed precise c) error estimate (1σ): <parameter name>:<error value> d) error interval: <parameter name> ± <error value> EXAMPLE error estimate (1σ): $a : 1\ 250$, $f^{-1} : 0{,}25$. If by reference, this specification element shall express the value(s) and error estimate(s) using the terminology found in the reference. These terms shall be enclosed in brackets ({}). Any parameter value that is not specified in the citation(s) shall be specified as in the “by value” case. An error estimate for b or for f^{-1} may be substituted in place of an error estimate for f.	
Date	The date the RD parameters were specified or published.	
References	The references (see 13.2.5).	

The RDs associated with physical objects are specified in Annex D. Table 7.10 is a directory of these RDs organized by type of ellipsoid. The semi-axis and radius parameters are unitless in position-space, but are bound to metre lengths when the RD is identified with the corresponding physical object-space constructed entity.

Table 7.10 — Physical RD specification table locations

Type of ellipsoid	RD table
Oblate ellipsoid	Table D.2
Sphere	Table D.3
Prolate ellipsoid	Table D.4
Tri-axial ellipsoid	Table D.5

Additional RDs associated with physical objects may be specified by registration in accordance with Clause 13.

7.2.5 RD binding

An RD is *bound* when the RD in position-space is identified with a corresponding constructed entity in object-space. In this context, a "constructed entity" is defined to mean an intrinsic, artificial, measured, or conceptual entity in object-space that is uniquely identifiable within the user's application domain. The term "corresponding" in this context means that each RD is bound to a constructed entity of the same geometric object type. That is, position-space points are bound to identified points in object-space, position-space directed lines to constructed lines or line segments in object-space, position-space directed curves to constructed curves or curve segments in object-space, position-space oriented planes to constructed planes or partial planes in object-space, and position-space oriented surfaces to constructed surfaces or partial surfaces in object-space.

When a curve or surface RD is bound, the radii of curvature on the corresponding constructed entity in object-space shall correspond to the radii of curvature in position-space. In this International Standard, in the case of physical objects, one unit in position-space corresponds to one metre in object-space. In the case of abstract objects, one unit in position-space corresponds to the designated length scale unit in the abstract object-space. In particular, the semi-axes of an ellipsoid RD shall correspond to the semi-axes of the constructed ellipsoid to which it is bound.

If the constructed entity of an RD binding is fixed in position with respect to object-space, then the RD binding shall be called an *object-fixed RD binding.* This definition assumes that the position of the constructed entity does not change in time by an amount significant for the accuracy and time scale of an application.

EXAMPLE 1 For points on the surface of the Earth, tectonic plate movements are insignificant for many applications.

EXAMPLE 2 An RD X_AXIS_3D is bound to the line segment from the centre of the Earth to the centre of the Sun. This RD binding is not an object-fixed RD binding with respect to the spatial object Earth.

Figure 7.2 illustrates two distinct bindings of a point RD. On the left, it is bound to a specific point in the abstract object-space of a CAD/CAM model. On the right, it is bound to a point in physical object-space that is on an object that has been manufactured from that CAD model.

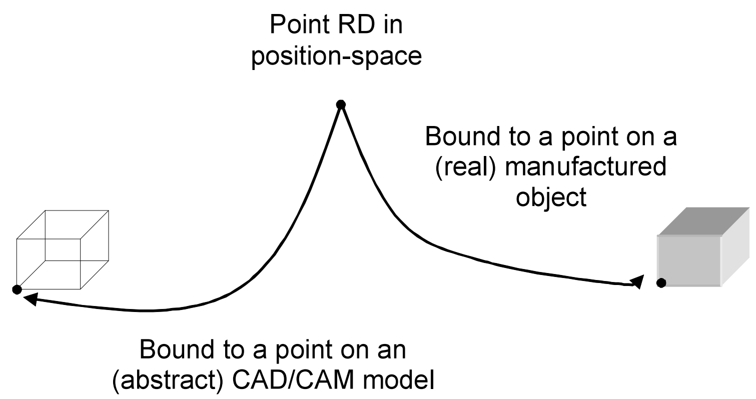

Figure 7.2 — An RD bound to an abstract object and to a real object

7.3 Normal embeddings of position-space into object-space

7.3.1 Normal embeddings

An embedding is a position-space model of object-space formed by a one-to-one function of positions in position-space to points in object-space. A *normal embedding* is an embedding that satisfies the following distance-preserving property:

A function $\boldsymbol{E}$ from position-space to object-space is *distance-preserving* if for any two positions $\boldsymbol{p}$ and $\boldsymbol{q}$ in position-space, the measured distance in object-space from $\boldsymbol{E}(\boldsymbol{p})$ to $\boldsymbol{E}(\boldsymbol{q})$ in metres is equal to the Euclidean distance $d(\boldsymbol{p}, \boldsymbol{q})$.

NOTE As a consequence of the normal distance-preserving property, a normal embedding is also a continuous function that preserves angles, area, and other geometric properties.

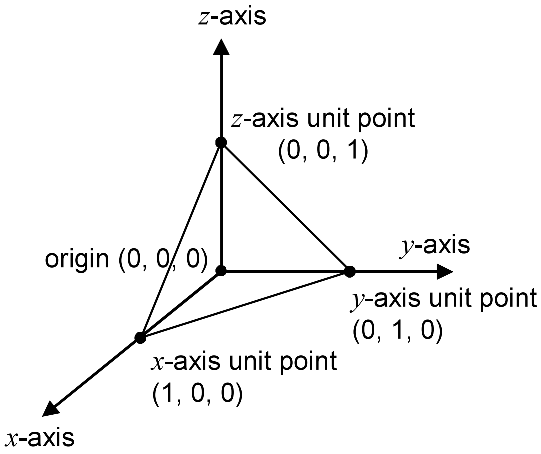

Figure 7.3 — A right-handed normal embedding[22]

In object-space, the point $\boldsymbol{E}(\mathbf{0})$ is called the *origin of the normal embedding* $\boldsymbol{E}$, and the point $\boldsymbol{E}(\boldsymbol{e}_1)$ is the *x-axis unit point* of the normal embedding $\boldsymbol{E}$. If the dimension of position-space is 2D or 3D, the point $\boldsymbol{E}(\boldsymbol{e}_2)$ is the *y-axis unit point* of the normal embedding $\boldsymbol{E}$. If the dimension of position-space is 3D, the point $\boldsymbol{E}(\boldsymbol{e}_3)$ is the *z-axis unit point* of the normal embedding $\boldsymbol{E}$.

A normal embedding of a 3D position-space is *right-handed* if the triangle formed by the directed curves connecting the three points, x-axis unit point, y-axis unit point, and z-axis unit point, in that sequence, has a clockwise orientation when viewed from the origin of the embedding. Otherwise, the embedding is *left-handed*. A right-handed normal embedding is illustrated in Figure 7.3. All 3D normal embeddings in this International Standard shall be right-handed.

7.3.2 Similarity transformations

A 3D object-space may have many normal embeddings of 3D position-space. Given two 3D normal embeddings $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ into the same object-space, one embedding can be expressed in terms of the other normal embedding. Given a position $(x, y, z)_{\mathrm{E2}}$ in position-space, the normal embedding $\boldsymbol{E}_2$ associates to it a unique point $\boldsymbol{p}$ in object-space. The normal embedding $\boldsymbol{E}_1$ uniquely associates some position $(x, y, z)_{\mathrm{E1}}$ to the same point $\boldsymbol{p}$. This association of $(x, y, z)_{\mathrm{E2}}$ to $(x, y, z)_{\mathrm{E1}}$ may be expressed as a similarity transformation from $\boldsymbol{E}_2$ to $\boldsymbol{E}_1$ (see Figure 4.2).

In general, $\boldsymbol{E}_2(\mathbf{0})$ may be displaced with respect to $\boldsymbol{E}_1(\mathbf{0})$ and the axes of the $\boldsymbol{E}_2$ normal embedding may also be rotated and/or differently scaled with respect to the axes of the $\boldsymbol{E}_1$ normal embedding (see Figure 7.4). The scale adjustment is needed to account for differing length scales in abstract object-space. In the case of physical object-space, scale factors close to 1,0 in value may be required to adjust for spatial distortions in empirically estimated data. This is addressed in 7.4.5. The same type of relationships also hold for 2D normal embeddings. *A similarity transformation* is defined as a transformation on position-space that performs a translation, rotation, and/or scaling operation.

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[22] The y-axis and z-axis are in the plane of the presentation, and the x-axis is directed generally towards the observer.

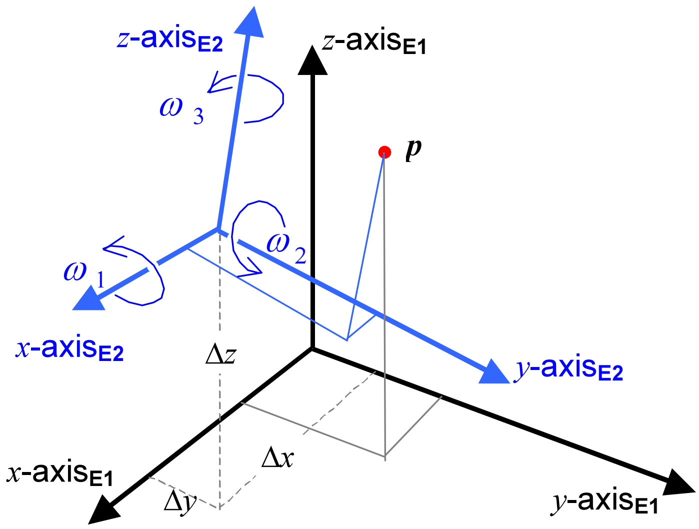


Figure 7.4 — 3D normal embedding relationships

7.3.3 Similarity transformation template

In both 2D and 3D position space, there are several ways to represent a similarity transformation. The particular representation of these operations may depend on the domain of a user application and on the types of physical and data models used to determine the appropriate similarity transformation. The number of parameters used in various representations, specified below, varies from zero parameters for the identity transformation to 13 parameters for one of the general matrix formulations.

A Similarity transformation template (STT) is a template format for specifying the parameters of a particular representation of a similarity transformation. The templates are labelled and coded and the set of templates is extensible by registration. The use of STTs adds flexibility and simplicity to the process of specifying an RT (see 7.4.5) or other similarity transformation. By using the appropriate STT, an RT can be specified with the original source data values for STT parameters. The elements of an STT specification are defined in Table 7.11. Standard STTs are specified in Tables 7.12 though 7.28. The STT specification elements include STT formulation and STT inverse formulation elements that define the mathematical formulation of the similarity transformation in terms of the STT parameter values and the position-space coordinate for a (source) point $(x, y, z)_{\mathrm{S}}$ and the corresponding transformed (target) point coordinate $(x, y, z)_{\mathrm{T}}$ in the 3D case, and $(x, y)_{\mathrm{S}}$ and $(x, y)_{\mathrm{T}}$ in the 2D case.

In the case of a similarity transformation that serves as an RT for an ORM, the constraint that one unit in object-space must have length one metre should disallow non-unit scaling. However, many local ERMs have RTs that are determined as a best fit to empirical data for which a very small scale adjustment provides an additional degree of freedom to reduce the residual error of the fit [RAPP2]. These small scale adjustments are permitted when specifying an RT for a local ORM.

Table 7.11 — STT specification elements

Element	Definition
STT label	The label for the STT (see 13.2.2).
STT code	The code for the STT (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Name(s)	The name or names given to this form of similarity transformation.
Description	A short description.
Dimension	The domain dimension indicated as "2D" or "3D".
STT parameters	Parameter symbols in a specified order with optional names and/or descriptions and units of measure (or unitless). "none" if there are no parameters.
STT constraints	Parametric constraints, if any, or "none".
STT formulation	An equation for the similarity transformation in terms of the STT parameters and the source position $\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S$ and target $\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T$ position.
STT inverse formulation	An equation for the inverse similarity transformation in terms of the STT parameters and the source and target positions.
Note(s)	Optional notes.
Reference(s)	Bibliographic reference(s) (see 13.2.5), or “none” if defined in this International Standard.

In some of the STT specifications below, the STT formulation and/or STT inverse formulation use notation for principal axis rotational operators. The notation and definition of those operators are defined in matrix form in Table 6.2. There are two conventions (see 6.2) in use for specifying the angle of rotation. Either the angle is measured from the starting position of a point to its rotated position, or it is measured from its rotated position to its starting position. The first convention is the position vector rotation (PVR) convention, and the second convention is the coordinate frame rotation (CFR) convention.

In Tables 7.12 through 7.28, the term *origin displacement*, if used, is defined as the target object-space point that corresponds to the source object-space origin.

Table 7.12 — STT identity transformation

Element	Definition
STT label	IDENTITY
STT code	1
Name(s)	identity transformation
Description	Identity transformation in three-dimensional object-space.
Dimension	3D
STT parameters	none
STT constraints	none

Element	Definition
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}}$
Note(s)	This STT is used for the object reference RT and other identity RTs.
Reference(s)	none

Table 7.13 — STT 2D identity transformation

Element	Definition
STT label	IDENTITY_2D
STT code	2
Name(s)	identity transformation
Description	Identity transformation in two-dimensional object-space.
Dimension	2D
STT parameters	none
STT constraints	none
STT formulation	$\begin{pmatrix} x \\ y \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} x \\ y \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \end{pmatrix}_{\mathrm{S}} = \begin{pmatrix} x \\ y \end{pmatrix}_{\mathrm{T}}$
Note(s)	This STT is used for the object reference RT and other identity RTs.
Reference(s)	none

Table 7.14 — STT translate

Element	Definition
STT label	TRANSLATE
STT code	3
Name(s)	translation of the origin
Description	Translated origin in three-dimensional object-space.
Dimension	3D

STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. Δz : the z-component of the origin displacement in metres.
STT constraints	none
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix}_S$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_T - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$
Note(s)	This is a common RT form for local ERMs.
Reference(s)	[83502T]

Table 7.15 — STT translate 2D

Element	Definition
STT label	TRANSLATE_2D
STT code	4
Name(s)	translation of the origin
Description	Translated origin in two-dimensional object-space.
Dimension	2D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres.
STT constraints	none
STT formulation	$\begin{pmatrix} x \\ y \end{pmatrix}_T = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}_S$
STT inverse formulation	$\begin{pmatrix} x \\ y \end{pmatrix}_S = \begin{pmatrix} x \\ y \end{pmatrix}_T - \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$
Note(s)	none
Reference(s)	none

Table 7.16 — STT position vector seven-parameter

Element	Definition
STT label	PV_7_PARAMETER
STT code	5

Element	Definition
Name(s)	seven-parameter transformation (PVR convention) simplified Helmert similarity transformation (PVR convention) Bursa-Wolf method (PVR convention)
Description	Helmert transformation using the Bursa-Wolf small angle approximation of the rotation matrix with angles in the PVR convention.
Dimension	3D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. Δz : the z-component of the origin displacement in metres. ω_1 : the x-axis PVR in radians. ω_2 : the y-axis PVR in radians. ω_3 : the z-axis PVR in radians. Δs : the scale difference from unity (unitless).
STT constraints	1) ω_1, ω_2 and, ω_3 are small rotations (magnitude less than 2x10^{-4}radians) in the PVR convention, 2) Δs is a small adjustment of scale ($\lvert\Delta s\rvert < 10^{-5}$).
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + (1+\Delta s)\begin{pmatrix} 1 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S = (1-\Delta s)\begin{pmatrix} 1 & \omega_3 & -\omega_2 \\ -\omega_3 & 1 & \omega_1 \\ \omega_2 & -\omega_1 & 1 \end{pmatrix}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}\right)$
Note(s)	1) This transformation widely used in geodesy with rotation angle values specified in arc second or milliarcsecond (mas) units. 2) The matrix in the formulation is the Bursa-Wolf small angle approximation of a rotation matrix. When multiplied with its transverse, the matrix product approximates the identity matrix with a matrix element-wise absolute error of 10^{-8} or less. The factor $(1-\Delta s)$ approximates $1/(1+\Delta s)$ with absolute error <10^{-10}. 3) The approximation uses the results that $\lvert\omega - \sin(\omega)\rvert < \omega^3/6$ and $\lvert 1-\cos(\omega)\rvert < \omega^2/2$ when ω is expressed in radians. When $\omega = 1''$, the error multiplied by the radius of the Earth is less than 75 μm. 4) An alternative form is $\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + \begin{pmatrix} 1+\Delta s & -\omega_3 & \omega_2 \\ \omega_3 & 1+\Delta s & -\omega_1 \\ -\omega_2 & \omega_1 & 1+\Delta s \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S.$ The difference between the two forms is negligible when the parameter constraints are met (10^{-8} magnitude error or less).

Element	Definition
Reference(s)	[RAPP2].

Table 7.17 — STT coordinate frame seven-parameter

Element	Definition
STT label	CF_7_PARAMETER
STT code	6
Name(s)	seven-parameter transformation (CFR convention) Helmert transformation (CFR convention) Bursa-Wolf method (CFR convention)
Description	Helmert transformation using the Bursa-Wolf small angle approximation of the rotation matrix, with angles in the CFR convention.
Dimension	3D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. Δz : the z-component of the origin displacement in metres. ω_1 : the x-axis CFR in radians. ω_2 : the y-axis CFR in radians. ω_3 : the z-axis CFR in radians. Δs : the scale difference from unity (unitless).
STT constraints	1) ω_1, ω_2 and, ω_3 are small rotations (magnitude less than 2x10^{-4} radians) in the CFR convention, 2) Δs is a small adjustment of scale ($\lvert\Delta s\rvert < 10^{-5}$).
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + (1+\Delta s)\begin{pmatrix} 1 & \omega_3 & -\omega_2 \\ -\omega_3 & 1 & \omega_1 \\ \omega_2 & -\omega_1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} = (1-\Delta s)\begin{pmatrix} 1 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 1 \end{pmatrix}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}\right)$

Element	Definition
Note(s)	1) This is a traditional transformation used in geodesy with rotation angle values often specified in arc second or milliarcsecond (mas) units. 2) The factor $(1-\Delta s)$ approximates $1/(1+\Delta s)$ with absolute error <10^{-10}. The matrix in the formulation is the Bursa-Wolf small angle approximation of a rotation matrix. When multiplied with its transverse, the matrix product approximates the identity matrix with a matrix element-wise absolute error of 10^{-8} or less. The approximation uses the results that $\lvert\omega-\sin(\omega)\rvert<\omega^3/6$ and $\lvert 1-\cos(\omega)\rvert<\omega^2/2$ when ω is expressed in radians. When $\omega=1''$, the error multiplied by the radius of the Earth is less than 75 μm. 3) An alternative form of the STT formulation is $\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + \begin{pmatrix} 1+\Delta s & \omega_3 & -\omega_2 \\ -\omega_3 & 1+\Delta s & \omega_1 \\ \omega_2 & -\omega_1 & 1+\Delta s \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}.$ The difference between the two forms is negligible when the parameter constraints are met (10^{-8} magnitude error or less).
Reference(s)	[RAPP2]

Table 7.18 — STT coordinate frame seven- plus three-parameter

Element	Definition
STT label	CF_7_PLUS_3_PARAMETER
STT code	7
Name(s)	Molodensky Badekas 10-parameter transformation (CFR convention) seven- plus three-parameter geometric transformation (CFR convention) Molodensky (7+3) model (CFR convention)
Description	A transformation between a local ORM and a global ORM with an origin displacement, and with both a small scale adjustment and small angle approximation of the rotation matrix, with angles in the CFR convention, and centred at the initial point (or datum origin) of the local ORM.
Dimension	3D

<table>
<tr><th>Element</th><th>Definition</th></tr>
<tr><td>STT parameters</td><td>Δx : the x-component of the origin displacement in metres.
Δy : the y-component of the origin displacement in metres.
Δz : the z-component of the origin displacement in metres.
ω_1: the x-axis CFR in radians.
ω_2 : the y-axis CFR in radians.
ω_3 : the z-axis CFR in radians.
Δs : the scale difference from unity (unitless).
x_0 : the x-component of the initial point in metres.
y_0 : the y-component of the initial point in metres.
z_0 : the z-component of the initial point in metres.</td></tr>
<tr><td>STT constraints</td><td>1) ω_1, ω_2 and, ω_3 are small rotations (magnitude less than 2x10^{-4} radians) in the CFR convention,
2) Δs is a small adjustment of scale ($|\Delta s| < 10^{-5}$).</td></tr>
<tr><td>STT formulation</td><td>$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_S + \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + \begin{pmatrix} \Delta s & \omega_3 & -\omega_2 \\ -\omega_3 & \Delta s & \omega_1 \\ \omega_2 & -\omega_1 & \Delta s \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}_S$$</td></tr>
<tr><td>STT inverse formulation</td><td>$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S = (1-\Delta s) \begin{pmatrix} 1 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 1 \end{pmatrix} \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} \right) + (1-\Delta s) \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}_S$$</td></tr>
<tr><td>Note(s)</td><td>1) If $(x_0, y_0, z_0) = (0,0,0)$, this STT is equivalent to the CF_7_PARAMETER STT.
2) The parameters are fit to empirical data so as to minimize residual error. The intent of using this formulation with the initial point of the local ORM is to produce a smaller residual error.
3) The factor $(1-\Delta s)$ approximates $1/(1+\Delta s)$ with absolute error <10^{-10} if $|\Delta s| < 10^{-5}$.
The matrix in the formulation is the Bursa-Wolf small angle approximation of a rotation matrix. When multiplied with its transverse, the matrix product approximates the identity matrix with a matrix element-wise absolute error of 10^{-8} or less.
The approximation uses the results that $|\omega - \sin(\omega)| < \omega^3/6$ and $|1-\cos(\omega)| < \omega^2/2$ when ω is expressed in radians. When $\omega = 1''$, the error multiplied by the radius of the Earth is less than 75 μm.
4) The inverse formulation is approximate as it drops terms of magnitude 10^{-8} or less. (See notes for CF_7_PARAMETER STT.)</td></tr>
<tr><td>Reference(s)</td><td>[GN72]</td></tr>
</table>

Table 7.19 — STT rotate scale translate

Element	Definition
STT label	ROTATE_SCALE_TRANSLATE
STT code	8
Name(s)	general similarity transformation in 3D
Description	A scaled rotation matrix followed by a translation.
Dimension	3D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. Δz : the z-component of the origin displacement in metres. $\left.\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right\}$: matrix $\boldsymbol{M}$ coefficients (unitless). s : the scale factor (unitless).
STT constraints	1) $\boldsymbol{M}$ is a rotation matrix: $\boldsymbol{M}^{-1} = \boldsymbol{M}^{\mathrm{T}}$ and $\det(\boldsymbol{M}) = 1$, 2) $s > 0$
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + s\boldsymbol{M} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$ where: $\boldsymbol{M} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} = \frac{1}{s}\boldsymbol{M}^{-1}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}\right)$
Note(s)	1) The most general form of a 3D similarity transformation. 2) Used in computer graphics applications.
Reference(s)	none

Table 7.20 — STT rotate scale translate 2D

Element	Definition
STT label	ROTATE_SCALE_TRANSLATE_2D
STT code	9
Name(s)	general similarity transformation in 2D
Description	A scaled rotation matrix followed by a translation.

Element	Definition
Dimension	2D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. $\left.\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}\right\}$: matrix $\boldsymbol{M}$ coefficients (unitless). s : the scale factor (unitless).
STT constraints	1) $\boldsymbol{M}$ is a rotation matrix: $\boldsymbol{M}^{-1} = \boldsymbol{M}^{\mathrm{T}}$ and $\det(\boldsymbol{M}) = 1$, 2) $s > 0$
STT formulation	$\begin{pmatrix} x \\ y \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}_{\mathrm{S,T}} + s\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix}_{\mathrm{S}}$ where: $\boldsymbol{M} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$
STT inverse formulation	$\begin{pmatrix} x \\ y \end{pmatrix}_{\mathrm{S}} = \frac{1}{s}\boldsymbol{M}^{-1}\left(\begin{pmatrix} x \\ y \end{pmatrix}_{\mathrm{T}} - \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}\right)$
Note(s)	1) The most general form of a 2D similarity transformation. 2) Used in computer graphics applications.
Reference(s)	none

Table 7.21 — STT homogeneous matrix 4x4

Element	Definition
STT label	HOMOGENEOUS_MATRIX_4X4
STT code	10
Name(s)	4x4 homogeneous transformation homogeneous transformation matrix
Description	A scaled rotation and translation in a 4x4 matrix form.
Dimension	3D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. Δz : the z-component of the origin displacement in metres. $\left.\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right\}$: sub-matrix $\boldsymbol{M}$ coefficients (unitless).

Element	Definition
STT constraints	The sub-matrix $\boldsymbol{M} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is a scaled rotation matrix: $\det(\boldsymbol{M}) > 0$ and $\boldsymbol{M}^{-1} = \frac{1}{\det(\boldsymbol{M})^2}\boldsymbol{M}^{\mathrm{T}}$.
STT formulation(s)	$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \Delta x \\ a_{21} & a_{22} & a_{23} & \Delta y \\ a_{31} & a_{32} & a_{33} & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}_{\mathrm{S}} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & -u \\ b_{21} & b_{22} & b_{23} & -v \\ b_{31} & b_{32} & b_{33} & -w \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}_{\mathrm{T}}$ where: $\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \boldsymbol{M}^{-1}$, and $\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \boldsymbol{M}^{-1}\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$.
Note(s)	Used in robotics and computer graphics rendering applications.
Reference(s)	[OPGL]

Table 7.22 — STT homogeneous matrix 3x3 2D

Element	Definition
STT label	HOMOGENEOUS_MATRIX_3X3_2D
STT code	11
Name(s)	3x3 homogeneous transformation
Description	A scaled rotation and translation in a 3x3 matrix form.
Dimension	2D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. $\left.\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}\right\}$: sub-matrix $\boldsymbol{M}$ coefficients.
STT constraints	The sub-matrix $\boldsymbol{M} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is a scaled rotation matrix: $\det(\boldsymbol{M}) > 0$ and $\boldsymbol{M}^{-1} = \frac{1}{\det(\boldsymbol{M})^2}\boldsymbol{M}^{\mathrm{T}}$

Element	Definition
STT formulation	$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_T = \begin{pmatrix} a_{11} & a_{12} & \Delta x \\ a_{21} & a_{22} & \Delta y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_S$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_S = \begin{pmatrix} b_{11} & b_{12} & -u \\ b_{21} & b_{22} & -v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_T$ where: $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \boldsymbol{M}^{-1}$, and $\begin{pmatrix} u \\ v \end{pmatrix} = \boldsymbol{M}^{-1} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$.
Note(s)	Used in computer graphics rendering applications.
Reference(s)	none

Table 7.23 — STT coordinate frame *x*-*y*-*z* rotate scale translate

Element	Definition
STT label	CF_XYZ_ROTATE_SCALE_TRANSLATE
STT code	12
Name(s)	x-y-z principal axes rotation, scaling, and translation (CFR convention)
Description	General 3D similarity transformation with principal axis rotations in axis order x-y-z and rotations in the CFR convention.
Dimension	3D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. Δz : the z-component of the origin displacement in metres. ω_1 : the x-axis CFR in radians. ω_2 : the y-axis CFR in radians. ω_3 : the z-axis CFR in radians. Δs : the scale difference from unity (unitless).
STT constraints	$\Delta s > -1$
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + (1+\Delta s)\, \boldsymbol{\Omega}_X(\omega_1) \circ \boldsymbol{\Omega}_Y(\omega_2) \circ \boldsymbol{\Omega}_Z(\omega_3) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_S$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_S = \left(\frac{1}{(1+\Delta s)} \right) \boldsymbol{\Omega}_Z(-\omega_3) \circ \boldsymbol{\Omega}_Y(-\omega_2) \circ \boldsymbol{\Omega}_X(-\omega_1) \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_T - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} \right)$
Note(s)	1) A generalization of the Helmert transformation allowing large rotations. 2) This the form of 3D RT specification in the first edition of the SRM.
Reference(s)	none

Table 7.24 — STT position vector x-y-z rotate translate

Element	Definition
STT label	PV_XYZ_ROTATE_TRANSLATE
STT code	14
Name(s)	Tait-Bryan rotation and translation world to entity transformation
Description	A principal axis rotation in axis order x-y-z in the PVR convention, with space-fixed coordinates, followed by a translation (see 6.4.4.4).
Dimension	3D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. Δz : the z-component of the origin displacement in metres. ω_1 : the x-axis PVR in radians. ω_2 : the y-axis PVR in radians. ω_3 : the z-axis PVR in radians.
STT constraints	none
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + \boldsymbol{R}_Z(\omega_3) \circ \boldsymbol{R}_Y(\omega_2) \circ \boldsymbol{R}_X(\omega_1) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} = \boldsymbol{R}_X(-\omega_1) \circ \boldsymbol{R}_Y(-\omega_2) \circ \boldsymbol{R}_Z(-\omega_3) \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} \right)$
Note(s)	1) A non-scaled generalization of the Helmert transformation allowing large rotations. 2) The angle parameters are known by the following names: ω_1 : heading or yaw or azimuth ω_2 : pitch or elevation angle ω_3 : roll or bank or tilt 3) This is equivalent to a world to entity space-fixed rotation in the IEEE 1278.1-1995 Standard.
Reference(s)	[DIS95]

Table 7.25 — STT position vector z rotate translate

Element	Definition
STT label	PV_Z_ROTATE_TRANSLATE
STT code	16

Element	Definition
Name(s)	z-axis rotation and origin translation longitudinal shift
Description	z-axis rotation in the PVR convention with origin translation. This transformation is used for Earth oblate ellipsoid ORMs with non-Greenwich prime meridians.
Dimension	3D
STT parameters	Δx : the x-component of the origin displacement in metres. Δy : the y-component of the origin displacement in metres. Δz : the z-component of the origin displacement in metres. ω : the z-axis PVR in radians.
STT constraints	none
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} + \boldsymbol{R}_Z(\omega) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} = \boldsymbol{R}_Z(-\omega) \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} \right)$
Note(s)	The single z-axis rotation is useful for the specification of the RT for an Earth ORM with non-Greenwich prime meridians.
Reference(s)	none

Table 7.26 — STT coordinate frame Z rotate

Element	Definition
STT label	CF_Z_ ROTATE
STT code	17
Name(s)	z-axis rotation (CFR convention)
Description	z-axis rotation in the CFR convention.
Dimension	3D
STT parameters	ω : the z-axis CFR in radians.
STT constraints	none
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \boldsymbol{\Omega}_Z(\omega) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} = \boldsymbol{\Omega}_Z(-\omega) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}}$
Note(s)	none
Reference(s)	none

Table 7.27 — STT position vector y-z rotate

Element	Definition
STT label	PV_YZ_ROTATE
STT code	18
Name(s)	y-z-axis rotation (PVR convention)
Description	y-z-axis rotation in the PVR convention.
Dimension	3D
STT parameters	ω_2 : the y-axis PVR in radians. ω_3 : the z-axis PVR in radians.
STT constraints	none.
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \boldsymbol{R}_Z(\omega_3) \circ \boldsymbol{R}_Y(\omega_2) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} = \boldsymbol{R}_Y(-\omega_2) \circ \boldsymbol{R}_Z(-\omega_3) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}}$
Note(s)	This STT is useful in RT specifications for celestiomagnetic ORMs (See 7.5.8).
Reference(s)	none

Table 7.28 — STT coordinate frame x-z rotate

Element	Definition
STT label	CF_XZ_ROTATE
STT code	19
Name(s)	x-z-axis rotation (CFR convention)
Description	x-z-axis rotation in the CFR convention.
Dimension	3D
STT parameters	ω_1: the x-axis CFR in radians. ω_3 : the z-axis CFR in radians.
STT constraints	none.
STT formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}} = \boldsymbol{\Omega}_X(\omega_1) \circ \boldsymbol{\Omega}_Z(\omega_3) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$
STT inverse formulation	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} = \boldsymbol{\Omega}_Z(-\omega_3) \circ \boldsymbol{\Omega}_X(-\omega_1) \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{T}}$

Element	Definition
Note(s)	This STT is useful in RT specifications for celestiomagnetic ORMs (See 7.5.8).
Reference(s)	none

7.4 Object reference model

7.4.1 Introduction

A set of bound RDs can be selected so as to be compatible with only one normal embedding. In this way, a set of bound RDs with properly constrained relationships can specify a unique normal embedding. Such a constrained set of bound RDs is called an object reference model. Some object reference models use a set of RDs that model application-specific geometric aspects of the object-space. Of particular interest are object reference models that include an oriented surface RD that models a surface significant to the object (see 7.4.2).

A relationship between two or more bound RDs needed to ensure compatibility with a normal embedding is termed a binding constraint (see 7.4.3). Object reference models that use the same set of RD primitives and the same binding constraints are abstracted in the notion of an object reference model template. Object reference model templates provide a uniform method of object reference model specification. If the bound RDs of an object reference model are compliant with the RD set and binding constraints of a particular object reference model template, then the object reference model is said to *realize* that template (see 7.4.4).

A set of standardized object reference model templates are defined in this International Standard (see 7.4.5). Realizations of these templates are specified in Annex E.

7.4.2 ORM

A normal embedding and an RD binding are *compatible* if the normal embedding image of the RD primitive is coincident with the points (and direction or orientation, as applicable) of the constructed entity of the RD binding.

EXAMPLE 1 The constructed point in object-space to which RD ORIGIN_3D is bound is the origin of a normal embedding if, and only if, that normal embedding is compatible with the RD binding.

EXAMPLE 2 The directed line constructed in object-space to which RD X_AXIS_3D is bound is the locus of the x-axis image under a compatible normal embedding, and similarly for other axis RDs.

An *object reference model* (ORM) for a spatial object is a set of bound RDs for which there exists exactly one normal embedding that is compatible with each bound RD in the set. In the 3D case, this unique embedding shall also be right-handed.

An ORM is *object-fixed* if each of its RD bindings is object-fixed, otherwise it is called *object-dynamic*. The object-fixed definition assumes that the object itself is not changing in time by an amount significant for the accuracy and time scale of an application. The normal embedding determined by an ORM is, correspondingly, either an *object-fixed embedding* or an *object-dynamic embedding*.

EXAMPLE 3 The Sun and the gas giants Jupiter, Saturn, Uranus, and Neptune are not rigid. The ORM specified for the Sun uses RD bindings defined in part by ephemeris and is thus object-dynamic. In the case of the ORMs specified for the gas giants, the object for binding is the magnetic field of the planet, thus these ORMs are object-fixed.

An ORM is often selected to contain an RD of category oriented surface that corresponds to a physical or conceptual surface significant to the modelled spatial object. An RD is chosen and its position with respect to the object is bound so that the RD instance is a “best fit” to the object in some application-specific sense. In particular, if the RD surface is "fitted" to a specific part of the object surface, the ORM is called a *local model*. If the RD is selected to best fit the entire surface, the ORM is called a *global model*.

An ORM may also contain an RD for the purpose of providing a CS binding parameter (see 8.3.2.2). In particular the radius of a sphere RD or the semi-axis values of an oblate ellipsoid RD may be used for this purpose.

An *Earth reference model* (ERM) is an ORM for which the spatial object is the Earth.

EXAMPLE 4 If the object is a planet, an ORM containing an oblate ellipsoid RD is usually selected to model all or part of the general shape of the planet.

7.4.3 Binding constraint

In an ORM, the existence of a unique and compatible normal embedding depends on establishing certain geometric relationships, called binding constraints, when the RDs are bound.

A *binding constraint* is a relationship in object-space between the constructed entities of two or more bound RDs, or a size relationship between a reference datum primitive and its corresponding constructed entity. Binding constraint relationships used in this International Standard include:

a) containment of a point in a curve or a surface;

b) containment of a curve in a surface;

c) coincidence of a line with an axis of symmetry of a surface;

d) in the 3D case, the right-handedness of sets of directed lines or oriented planes; or

e) distance measurement in object-space between points.

In this International Standard every abstract object-space has an associated length scale. The term "(scaled) metres" in a binding constraint definition shall mean "metres" in the case of physical objects and shall mean "length scaled to metres with respect to the length scale of object-space" in the case of abstract objects.

7.4.4 ORM template

ORMs that have the same set of RD primitives and that have the same binding constraints are abstracted in the notion of an ORM template. An ORM template specifies how certain sets of RD primitives may be bound to ensure that the resulting set of bound RDs forms an ORM. An ORM template can be used to conveniently specify multiple ORMs.

An *ORM template* (ORMT) is a set of RDs and binding constraints such that, for a given object-space, whenever the RDs in the set are bound in compliance with the set of binding constraints, then that bound set of RDs forms an ORM.

An ORM is a *realization* of an ORMT if

1) the RDs of the ORM match the RD set of the ORMT, and

2) the RD bindings of the ORM are compliant with the binding constraints of the ORMT.

This International Standard specifies a set of ORMTs for 2D and 3D position-space in Tables 7.31 and 7.32. The specification elements are defined in Table 7.29. Additional ORMTs may be registered in accordance with Clause 13. Table 7.30 is a directory of ORMT specification tables.

Table 7.29 — ORMT specification elements

Element		Definition
ORMT label		The label for the ORMT (see 13.2.2).
ORMT code		The code for the ORMT (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
ORMT specification	**Description**	A description of an ORM realization of the template.
	RD set	A list of RDs in the set.
	Binding constraints	Binding constraints.
	Notes	Optional notes.

Table 7.30 — ORMT specification directory

Position-space dimension	Table number
2D	Table 7.31
3D	Table 7.32

Table 7.31 — 2D ORMT specifications

ORMT label	ORMT code	ORMT specification
BI_AXIS_ORIGIN_2D	1	**Description:** x- and y-axes determined by directed perpendicular lines passing through the origin. **RD set:** RD 1) RD ORIGIN_2D RD 2) RD X_AXIS_2D RD 3) RD Y_AXIS_2D **Binding constraints:** BC 1) The constructed point bound to RD 1 shall be contained in the constructed directed line bound to RD 2 and the constructed directed line bound to RD 3. BC 2) The constructed directed lines bound to RD 2 and RD 3 shall be perpendicular. **Notes:** 1) The constructed point bound to RD 1 determines the origin of the normal embedding. 2) The perpendicular directed lines passing through the origin uniquely determine the x-axis and y-axis of the normal embedding.

Table 7.32 — 3D ORMT specifications

ORMT label	ORMT code	ORMT specification
SPHERE	2	**Description:** 3D sphere with designated directional axis and xz-plane. **RD set:** RD 1) The sphere RD with radius r. RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D **Binding constraints:** BC 1) The constructed directed line bound to RD 2 shall contain the centre of the constructed sphere bound to RD 1. BC 2) The constructed plane bound to RD 3 shall contain the constructed directed line bound to RD 2. BC 3) The radius of the constructed sphere bound to RD 1 shall be r (scaled) metres. **Notes:** 1) The centre of the constructed sphere bound to RD 1 determines the origin of the normal embedding. 2) The constructed directed line bound to RD 2 passing through the origin of the normal embedding uniquely determines the z-axis of the normal embedding. 3) The plane through the origin of the normal embedding perpendicular to the z-axis of the normal embedding determines the xy-plane of the normal embedding. The constructed plane bound to RD 3 determines the xz-plane of the normal embedding. The intersection of the constructed xz-plane with the xy-plane is the locus of the x-axis of the normal embedding. The positive side of the xz-plane is designated in the binding and together with the direction of the z-axis of the normal embedding determines the direction of the x-axis of the normal embedding. 4) The line perpendicular to the xz-plane of the normal embedding through the origin of the embedding determines the locus of the y-axis of the normal embedding. Its direction is determined by the right-handedness of the normal embedding. 5) The distance binding constraint BC 3 is required for compatibility with a normal embedding.

ORMT label	ORMT code	ORMT specification
OBLATE_ELLIPSOID	3	**Description:** Oblate ellipsoid with designated minor axis direction and xz-plane. **RD set:** RD 1) The oblate ellipsoid RD with major semi-axis a and minor semi-axis b RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D **Binding constraints:** BC 1) The spatial plane RD 3 shall contain the minor axis of the spatial oblate ellipsoid RD 1. BC 2) The spatial z-axis RD 2 shall coincide with the minor axis of the spatial oblate ellipsoid RD 1. BC 3) The RD 1 length of the spatial major semi-axis shall be a (scaled) metres and the length of the spatial minor semi-axis shall be b (scaled) metres. **Notes:** 1) The centre of the constructed oblate ellipsoid bound to RD 1 determines the origin of the normal embedding. 2) The constructed directed line bound to RD 2 passing through the origin (as required by BC 1) uniquely determines the z-axis of the normal embedding. 3) The z-axis of the normal embedding determines the xy-plane as the perpendicular plane through the origin of the normal embedding. BC 2 requires the bound xz-plane to contain the z-axis. The line formed by the intersection of the bound xz-plane with the xy-plane is the x-axis line. The positive side of the constructed plane bound to RD 3 determines the xz-plane and together with the direction of the z-axis determines the direction of the x-axis. 4) The y-axis is determined by the required right-handedness of the normal embedding. 5) The distance binding constraint BC 3 is required for compatibility with a normal embedding.

ORMT label	ORMT code	ORMT specification
PROLATE_ELLIPSOID	4	**Description:** 3D prolate ellipsoid with designated major axis direction and xz-plane. **RD set:** RD 1) The prolate ellipsoid RD with minor semi-axis a and major semi-axis b. RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D **Binding constraints:** BC 1) The spatial plane shall contain the major axis of the spatial prolate ellipsoid. BC 2) The spatial z-axis shall coincide with the major axis of the spatial prolate ellipsoid. BC 3) The length of the spatial major semi-axis shall be b (scaled) metres and the length of the spatial minor semi-axis shall be a (scaled) metres.
TRI_AXIAL_ELLIPSOID	5	**Description:** 3D tri-axial ellipsoid with designated z-axis direction and xz-plane. **RD set:** RD 1) The tri-axial ellipsoid RD with x-semi-axis a, y-semi-axis b, and z-semi-axis c. RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D **Binding constraints:** BC 1) The spatial plane shall contain the z-axis of the spatial tri-axial ellipsoid. BC 2) The spatial z-axis shall coincide with the z-axis of the spatial tri-axial ellipsoid. BC 3) The length of the spatial x-semi-axis shall be a (scaled) metres, spatial y-semi-axis shall be b (scaled) metres, and the length of the spatial z-semi-axis shall be c (scaled) metres.

ORMT label	ORMT code	ORMT specification
BI_AXIS_ORIGIN_3D	6	**Description:** x- and z-axes determined by directed perpendicular lines passing through the origin. **RD set:** RD 1) RD ORIGIN_3D RD 2) RD Z_AXIS_3D RD 3) RD X_AXIS_3D **Binding constraints:** BC 1) The constructed point bound to RD 1 shall be contained in the constructed directed line bound to RD 2 and the constructed directed line bound to RD 3. BC 2) The constructed directed lines bound to RD 2 and RD 3 shall be perpendicular. **Notes:** 1) The constructed point bound to RD 1 determines the origin of the normal embedding. 2) The perpendicular directed lines passing through the origin uniquely determine the z-axis and x-axis of the normal embedding. 3) The y-axis is determined by the required right-handedness of the normal embedding.

ORMT label	ORMT code	ORMT specification
SPHERE_ORIGIN	7	**Description:** Sphere with two directed perpendicular lines passing through the centre of the sphere. **RD set:** RD 1) RD ORIGIN_3D RD 2) RD Z_AXIS_3D RD 3) RD X_AXIS_3D RD 4) The sphere RD with radius r. **Binding constraints:** BC 1) The constructed point bound to RD 1 shall be contained in the constructed directed line bound to RD 2 and the constructed directed line bound to RD 3. BC 2) The constructed directed lines bound to RD 2 and RD 3 shall be perpendicular. BC 3) The centre of the constructed sphere bound to RD 4 shall be coincident with the point bound to RD 1. BC 4) The radius of the constructed sphere bound to RD 4 shall be r (scaled) metres. **Notes:** 1) The point bound to RD 1 determines the origin of the normal embedding. 2) The perpendicular directed lines passing through the sphere centre uniquely determine the z-axis and x-axis of the normal embedding. 3) The y-axis is determined by the required right-handedness of the normal embedding. 4) The distance binding constraint BC 4 is required for compatibility with a normal embedding. 5) The sphere RD is included to provide a CS binding parameter (radius). See 7.4.2.

ORMT label	ORMT code	ORMT specification
OBLATE_ELLIPSOID-_ORIGIN	8	**Description:** Oblate ellipsoid with designated centre, minor axis direction and xz-plane. **RD set:** RD 1) RD ORIGIN_3D RD 2) RD Z_AXIS_3D RD 3) RD XZ_PLANE_3D RD 4) The oblate ellipsoid RD with major semi-axis a and minor semi-axis b. **Binding constraints:** BC 1) The constructed point bound to RD 1 shall be contained in the constructed directed line bound to RD 2. BC 2) The spatial plane RD 3 shall contain the constructed directed line bound to RD 2. BC 3) The minor axis of the spatial oblate ellipsoid RD 4 shall be coincident with the directed line bound to RD 2 and the ellipsoid centre shall coincide with the point bound to RD 1. BC 4) The RD 4 length of the spatial major semi-axis shall be a (scaled) metres and the length of the spatial minor semi-axis shall be b (scaled) metres. **Notes:** 1) The centre of the constructed sphere bound to RD 1 determines the origin of the normal embedding. 2) The constructed directed line bound to RD 2 passing through the origin uniquely determines the z-axis of the normal embedding. 3) The z-axis of the normal embedding determines the xy-plane as the perpendicular plane through the origin of the normal embedding. BC 2 requires the bound xz-plane to contain the z-axis. The line formed by the intersection of the bound xz-plane with the xy-plane is the x-axis line. The positive side of the constructed plane bound to RD 3 determines the xz-plane and together with the direction of the z-axis determines the direction of the x-axis. 4) The y-axis is determined by the required right-handedness of the normal embedding. 5) The distance binding constraint BC 4 is required for compatibility with a normal embedding. 6) The oblate ellipsoid RD is included to provide CS binding parameters (major and minor semi-axes). See 7.4.2.

ORMT label	ORMT code	ORMT specification
TRI_PLANE	9	**Description:** Origin determined by the intersection of three planes. **RD set:** RD 1) RD XZ_PLANE_3D RD 2) RD XY_PLANE_3D RD 3) RD YZ_PLANE_3D **Binding constraints:** BC 1) The spatial planes shall be pair-wise perpendicular. BC 2) The collective formation of the three planes shall be right-handed. **Notes:** 1) The intersection of all three planes RD 1, RD 2, and RD 3 determine the origin of the normal embedding. 2) The intersection of the xz-plane RD 1 and the xy-plane RD 2 determine the line of the x-axis. 3) The positive side designations of the two planes RD 1 and RD 2 together with the right-handedness requirement determines the positive x-axis direction. The directed line and the origin point determine the x-axis of the normal embedding. 4) The z- and y-axes of the normal embedding are similarly determined. 5) BC 2 is required for compatibility with the right-handedness requirement.

The specification of an ORMT does not determine a normal embedding. A normal embedding is determined when an ORMT is realized as an ORM.

The methods and techniques of binding the RDs of an ORMT realization draw from disciplines ranging from geometry to astronomy, surveying, geophysics, and satellite geodesy. In general, there are many ways to realize an ORMT for a spatial object. Techniques and methodologies for binding RD components are outside of the scope of this International Standard. The ORM concept is designed to be general enough to encompass these many application domains. As an illustration of the generality of this concept, the following Example outlines a method used in geodesy to define the RD bindings of an ORMT OBLATE_ELLIPSOID realization.

EXAMPLE The North American Datum 1927 may be specified as a realization of the ORMT OBLATE_ELLIPSOID (see Table 7.32). The oblate ellipsoid RD component is RD CLARKE 1866 in Table D.2. The binding of this RD and the other two RD components in the Earth object-space may be defined as follows:

a) the direction of the RD Z_AXIS_3D is identified as the north direction of the Earth’s rotational axis,

b) a position (latitude 39°13’26,686”N) on the oblate ellipsoid RD is identified to a position in the Earth object-space (Meades Ranch, Kansas, US),

c) the direction of the surface normal to the ellipsoid at the identified point is specified (ξ = -1,32" η = 1,93"), and

d) the direction of the positive xz-plane normal is indirectly determined by specifying the longitude of Meades Ranch (98°32’30,506”W).

Items a) through c) determine a unique oblate ellipsoid surface in the object-space of the Earth. The equatorial plane of the oblate ellipsoid is (by compatibility with the oblate ellipsoid surface generating function) the xy-plane, and its intersection with the oblate ellipsoid axis of rotation determines the origin point and the z-axis. Specification (d) together with the origin and the xy-plane determine the x-axis. Since there is one, and only one, oriented yz-plane that is both perpendicular to the xz-plane and right-handed compatible, the right-handed normal embedding is uniquely determined (see Figure 7.5).

This Example is based on a published specification of the North American Datum 1927. However, the methodology used to select the point in b) and determine the surface normal direction c) at that point was a complex process involving a mathematical best fit of the Clarke 1866 ellipsoid to a network of geodetic survey control points spanning the continental United States.

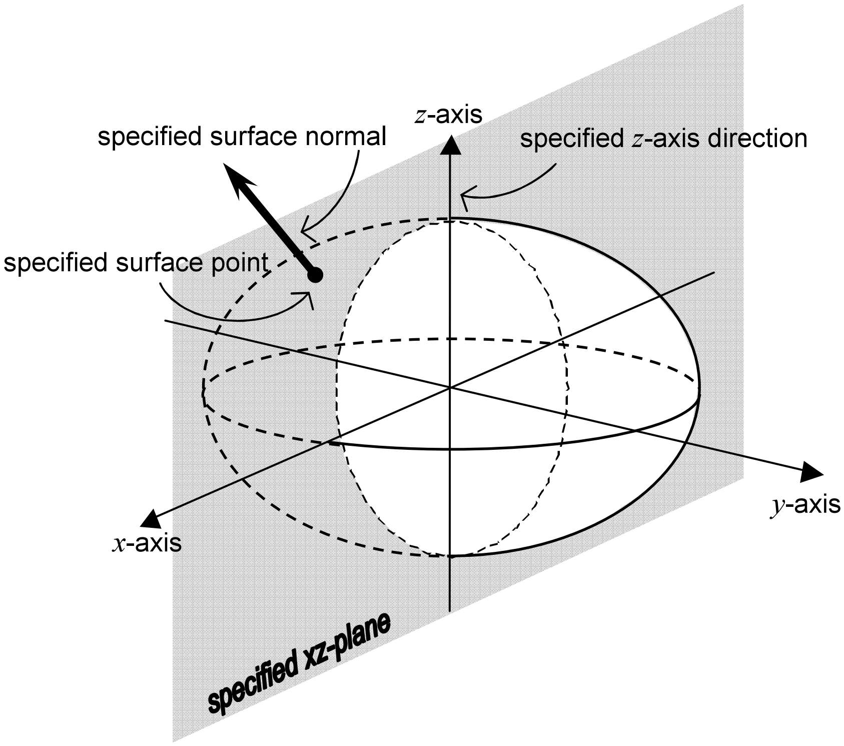


Figure 7.5 — Oblate ellipsoid ORMT binding

7.4.5 Standardized ORMs

The ORMs specified in this International Standard, are ORMT realizations. Standardized ORMs shall include:

a) a specification of the spatial object and optionally a region in object-space,

b) a specification of the ORM template,

c) a specification of the ellipsoid RD (if any), and

d) the binding year if the spatial object is a physical object.

A standardized ORM does not include a specification of the binding of its RD components. The binding specification is not directly needed for the scope of this International Standard. An ORM indirectly designates a unique normal embedding. A specification of an ORM normal embedding is important when two or more ORMs have been specified for the same spatial object. To inter-convert between spatial relationships with respect to two different normal embeddings, one normal embedding shall be expressed in terms of the other normal embedding by means of a similarity transformation, or both shall be expressed in terms of a third (reference) normal embedding by means of a similarity transformation for each ORM with respect to a third (reference) ORM.

If two or more object-fixed ORMs for the same object are specified (or registered), one of the ORMs shall be designated as the *reference ORM* for that object.

A *reference transformation* (RT) for an ORM is a similarity transformation from the ORM normal embedding to the normal embedding of the reference ORM for that object, ORM_R. The reference transformation for an ORM, ORM_S, shall be denoted by $\boldsymbol{H}_{\mathbf{SR}}$ (see Table 10.1).

For 3D ORMs, a reference transformation shall be specified as an STT together with the values that correspond to the STT parameters. In the case of dynamic ORMs (see 7.5), parameter values may be functions of time in a specified temporal coordinate system. Standardized object-fixed ORMs shall specify at least one reference transformation.

Some ERMs that are specified in Table E.5 are based on local geodetic datums. Historic local geodetic datums may exhibit distortions with respect to the reference ERM. In these cases, an RT specified by the ERM is an approximation based on empirical measurements. In some of these cases, similarity transformation parameters for approximations based on different sets of measurements and/or sub-regions appear as multiple RT entries in Table E.6. In those cases, the RT labels shall share a common prefix derived from the name of the related local geodetic datum.

NOTE 1 In some cases a scale adjustment is needed to account for differing length scales in abstract object-space. The STTs PV_7_PARAMETER, CF_7_PARAMETER, CF_7_PLUS_3_PARAMETER, ROTATE_SCALE_TRANSLATE, and CF_XYZ_ROTATE_SCALE_TRANSLATE provide either a scale factor s parameter or a scale adjustment parameter $(1+\Delta s)$. Theoretically, $s=1$ or $\Delta s=0$ for normal embeddings of a physical object-space. In practice, when the embeddings are indirectly determined by the RD bindings of an ORM, the determination of the parameters is an approximation that is the result of a mathematical best fit and the scale parameter (s or Δs) is used to provide an additional degree of freedom to achieve the desired accuracy in the region of interest. In the case of the object Earth, published values for Δs are typically one part per million in magnitude (10^{-6}) or smaller.

A directory of reference ORMs is provided in Table E.1. The reference ORM for the Earth is ORM WGS_1984. This ORM is an Earth-fixed global model. The actual binding definition of ORM WGS_1984 (see [83502T]) is a realization of ORMT BI_AXIS_ORIGIN_3D. However, for purposes of this International Standard, the RD WGS_1984 is associated with this ORM, so that its ORM specification is a realization of ORMT OBLATE_ELLIPSOID.

The elements of an ORM specification are defined in Table 7.33. Standardized ORMs are specified in Annex E. Additional ORMs may be registered in accordance with Clause 13.

Table 7.33 — ORM specification elements

Element	Definition
ORM label	The label for the ORM (see 13.2.2).
ORM code	The code for the ORM (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Published name	The name(s) given to the concept embodied in this ORM in the reference(s).
Reference ORM	The label of the reference ORM for this object. If this ORM is the reference ORM for this object, then this specification element shall contain the phrase “This is the reference ORM for” followed by the object name. If no object-fixed ORM specification exits, this specification element shall contain the string “none”.

Element	Definition
Binding information	Case: Object-fixed ORM for a physical object If the spatial object is a physical object, the date that the ORM RD components were bound in object-space. If the ORM is based on ORMT OBLATE_ELLIPSOID, OBLATE_ELLIPSOID_ORIGIN, SPHERE, or SPHERE_ORIGIN, a significant location contained in the x-positive xz-half-plane of the normal embedding shall be specified. In cases where the spatial object is the Earth, this location shall be understood to be Greenwich, UK, unless otherwise specified. Case: Dynamic ORM for a physical object If the ORM is based on ORMT BI_AXIS_ORIGIN_3D and if the ORM binding complies with a standard object binding rule set, the label of that object binding rule set (see 7.5). If the ORM is a time-fixed (object-fixed) instance of a dynamic ORM, the date that the ORM RD components were bound in object-space. Case: ORM for an abstract object The string “none”. All cases: Optional binding notes.
Region	The approximate subset of object-space to which the model applies, expressed as either a spatial extent or the description as specified in the reference.
ORMT label	The label of the ORM template for this ORM.
RD parameterization	The label of the ellipsoidal RD, if any; otherwise “n/a”.
References	The references (see 13.2.5), or “none” if defined in this International Standard.

For each object-fixed ORM there shall be specified one or more RTs that transform the ORM to the reference ORM of the ORM spatial object. The elements of an RT specification are defined in Table 7.34. Standardized RTs are specified in Annex E. Additional RTs may be registered in accordance with Clause 13. The standard ORM for an abstract space of a given dimension specifies only an identity RT. Graphical applications use *ad hoc* transformations to scale, rotate, and translate one abstract space with respect to another as needed in an application. The API (Clause 11) provides support for non-standardized RTs.

Table 7.34 — Reference transformation specification elements

Element	Definition
ORM label	The label of the standardized ORM that this RT transforms.
RT label	The label for the RT (see 13.2.2).
RT code	The code for the RT (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
RT region	A non-normative description of the extent and/or the spatial bounds of the region for which this reference transformation is applicable. Angles may be expressed in arc degrees (°) in order to avoid a loss of precision.
STT label	The label of the STT that is used to specify the $\boldsymbol{H}_{SR}$ transformation.

Element	Definition
STT parameters	The values of the STT parameters shall be specified by value or by reference (see 13.2.5) using the STT parameter symbols. If by value, the values of the STT parameters specifying the reference transformation $\boldsymbol{H}_{\text{SR}}$ (see Table 10.1) shall be specified. These values may be followed by a measurement/modelling error estimate expressed in one of the following forms: a) : assumed precise b) : σ<standard error> c) : ±<tolerance> d) no error information following a parameter value indicates that the error estimate is unknown or unattainable. EXAMPLE $\Delta x = 12 : \sigma 5,\ \Delta y = -133 : \pm 25,\ \Delta z = 0 : \text{assumed precise}$. If by reference, this specification element shall contain a citation(s) for the values of the STT parameters and error estimates. Terms appearing in the references that are cited for a value shall be enclosed in brackets ({ }). Any parameter value that is not specified in the citation(s) shall be specified as in the "by value" case. A dynamic $\boldsymbol{H}_{\text{SR}}(t)$ transformation may specify parameter values as functions of time. To avoid loss of precision, axis rotations angles (if applicable to the STT) may be expressed in arc seconds (") and, in cases of a large rotation, in arc degrees (°). NOTE The axis rotation angles are converted to radians when used in appropriate STT formulations.
Date published	The date that the RT was published.
References	The references (see 13.2.5), or "none" if defined in this International Standard.

RTs for standardized ORMs are specified in Annex E. In the annex E specification tables, the specification elements STT label and STT parameters are combined for legibility.

If a 3D ORM is the reference ORM of a spatial object, then it shall have an RT with the RT label containing the string "IDENTITY", and the RT shall be specified using the IDENTITY STT. These reference ORM identity RTs are labelled and coded to provide uniform treatment of all object-fixed ORMs in the API (see Clause 11).

If a 3D ORM is not the reference ORM of a spatial object, and an RT is the identity transformation by intent or design of the ORM, then the RT label shall contain the string "IDENTITY_BY_DEFAULT", and the RT shall be specified using the IDENTITY STT.

If a 3D ORM is not the reference ORM of a spatial object, and an RT of the ORM is empirically equivalent to the identity transformation, then the RT label shall contain the string "IDENTITY_BY_MEASUREMENT", and the RT shall be specified using the TRANSLATE STT with zero for the parameter values and appropriate measurement error estimates.

NOTE 2 In the case of Earth-fixed ERM bindings axis rotations are either zero, or are very small with the following exceptions:

a) Cases for which the xz-plane does not contain Greenwich, UK have relatively large ω_3 rotations.

b) The Earth-fixed approximations of celestiomagnetic ERMs have large ω_2 and ω_3 rotations (see "GEOMAGNETIC" entries in Table 7.50).

NOTE 3 A geodetic datum that is global or geocentric is used as a basis for establishing a geocentric SRF (see 8.5.2). If geocentric coordinates are associated to a 3D linear embedding, then such a datum is conceptually equivalent to an ERM. The WGS 84 global datum (see [83502T]) is conceptually equivalent to the ORM WGS_1984 in Table E.5.

7.5 Object binding rules for ORMT BI_AXIS_ORIGIN_3D realizations

7.5.1 Object binding rule set

ORMs for planets, satellites, and other celestial bodies include object-dynamic ORMs based on ORMT BI_AXIS_ORIGIN_3D. Many of these object-dynamic ORMs share common rules for the binding of ORMT BI_AXIS_ORIGIN_3D components based on physical characteristics and spatial arrangements of the applicable celestial bodies. To facilitate uniform specification of these ORMs, the concept of an object binding rule set for an ORMT BI_AXIS_ORIGIN realization is defined.

An *object binding rule set* (OBRS) for ORMT BI_AXIS_ORIGIN_3D shall be comprised of:

a) an object binding rule set name,

b) a label and code,

c) object restrictions that delineate the object-spaces for which the object binding rules apply, and

d) a set of object binding rules for the RD components of ORMT BI_AXIS_ORIGIN_3D,

where an *object binding rule* for an ORMT is an object-space specific restriction for the binding of a single RD in the RD set of the ORMT. The set of object binding rules (d) shall comply with the binding constraints of the ORMT.

The specification elements for an OBRS for ORMT BI_AXIS_ORIGIN_3D are defined in Table 7.35.

Table 7.35 — OBRS for ORMT BI_AXIS_ORIGIN_3D specification elements

Element	Definition
OBRS label	The label for the OBRS (see 13.2.2).
OBRS code	The code for the OBRS (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Short name	A descriptive name.
Object restrictions	A specification of a set of objects for which the object binding restrictions apply.
Object binding rules	A specification of the binding rules.
Figures	Zero or more figures that explain and illustrate the OBRS.
References	Zero or more references (see 13.2.5).

This International Standard provides a collection of OBRS specifications as identified in Table 7.36. Additional OBRS specifications may be registered in accordance with Clause 13.

Table 7.36 — OBRS for ORMT BI_AXIS_ORIGIN_3D specification directory

OBRS name	Table number
equatorial inertial	Table 7.37
solar ecliptic	Table 7.39
solar equatorial	Table 7.41
heliocentric Aries ecliptic	Table 7.43
heliocentric planet ecliptic	Table 7.45
heliocentric planet equatorial	Table 7.47
celestiomagnetic	Table 7.49
solar magnetic ecliptic	Table 7.51
solar magnetic dipole	Table 7.53

An OBRS name may be used to describe an ORM that is compliant with the named OBRS. Thus a "celestiomagnetic ORM" denotes an ORM realization of ORMT BI_AXIS_ORIGIN_3D for an object-space that satisfies the celestiomagnetic OBRS object restrictions and whose RD bindings comply with the celestiomagnetic OBRS object binding rule set.

Several OBRS specifications in Table 7.36 are described using one or more of the following terms: rotational northwards, vernal equinox, inertial direction, quasi-inertial direction, first point of Aries, and Aries true of date. These terms are defined as follows:

The term *rotational northwards*, as used with respect to a rotating object, shall mean in a direction making an acute angle with respect to the direction from the centre of the object to its rotational north pole.

The ecliptic plane and the plane containing the Sun and parallel to the equatorial plane of a planet intersect in a line. An *equinox* is one of the two points of intersection between this line and the orbit of the planet. The *vernal equinox* is the equinox at which the direction from the planet to the Sun begins to ascend to the northern side of the equatorial plane.

An *inertial direction* is a direction with respect to the universe that is time invariant. A *quasi-inertial direction* is a direction that changes relatively slowly with respect to the universe.

At any given epoch, the direction of the Sun at the vernal equinox is fixed with respect to the Milky Way and other (detectible) galaxies. In the case of the Earth, the *first point of Aries* is the direction from the vernal equinox to the Sun. The current first point of Aries is termed *Aries true of date*.

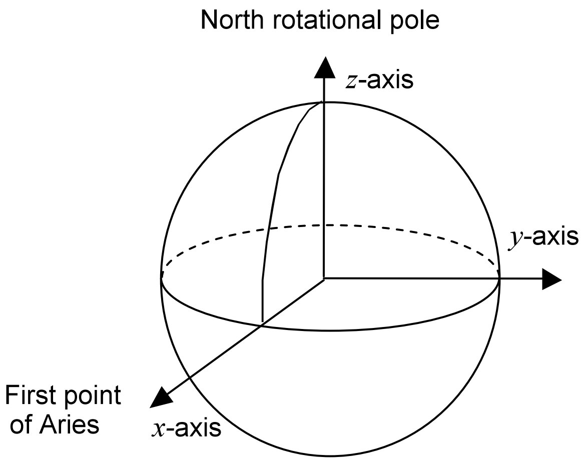
NOTE The effects of precession and nutation on the spin axis of a planet cause this direction to change over time. The change in direction of the first point of Aries is very slow (approximately one full rotation in 26 000 years).

EXAMPLE The first point of Aries at a given epoch is an inertial direction, and Aries true of date is a quasi-inertial direction.

7.5.2 Equatorial inertial

The *equatorial inertial OBRS* is specified in Table 7.37.

Table 7.37 — Equatorial inertial OBRS

Element	Value
OBRS label	EQUATORIAL_INERTIAL
OBRS code	1
Short name	equatorial inertial
Object restrictions	A planet in the solar system for which the ecliptic plane is distinct from the equatorial plane.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD X_AXIS_3D points in the direction of the Sun when the planet is at its vernal equinox. 3) The RD Z_AXIS_3D is parallel to the rotational axis and points in the direction of rotational northwards.
Figures	
References	[SEID]

The axis directions are quasi-inertial, but vary with respect to any object-fixed ORM for the planet.

In the case of ORM EARTH_INERTIAL_J2000r0, the International Earth Rotation Service (see [IERS]) specifies a very precise transformation to the ORM WGS_1984 reference embedding with a matrix whose coefficients represent the effects of polar motion, the Earth's rotation, nutation and precession[23].

EXAMPLE The *Greenwich sidereal hour angle* $\theta_{\text{GSH}}(t)$ is the angle in radians from the first point of Aries to the direction of the x-axis of ORM WGS_1984 Earth reference ORM. The Greenwich sidereal hour angle depends on the epoch of definition of the first point Aries and is a function of UTC time t elapsed from a given epoch. Approximations of this function are published in astronomical almanacs and other documents (see [SEID] or [USNOA]).

[23] For near-real-time orbit determination applications, Earth orientation parameters (polar motion and Earth rotation variations) that are needed to build the matrix are predicted values. Because the driving forces that influence polar motion and Earth rotation variations are difficult to characterize, these Earth orientation predictions are performed weekly [83502T].

Given an ERM in the equatorial inertial OBRS, if it is assumed that the ERM z-axis and ORM WGS_1984 z-axis are coincident, then the ERM RT is specified by STT CF_Z_ROTATE with dynamic parameter $\omega(t) = \theta_{\text{GSH}}(t)$.

Three equatorial inertial ERMs labelled ORM EARTH_INERTIAL_ARIES_1950, ORM EARTH_INERTIAL_J2000r0 and ORM EARTH_INERTIAL_ARIES_TRUE_OF_DATE, are specified in Table E.5. They are based on three determinations of the first point of Aries and the rotational axis. The first two have epoch-fixed inertial directions. ORM EARTH_INERTIAL_ARIES_TRUE_OF_DATE has a quasi-inertial x-axis direction that varies as a function of time.

Table 7.38 is a directory of the Earth and other celestial object equatorial inertial ORMs.

Table 7.38 — Equatorial inertial ORM directory

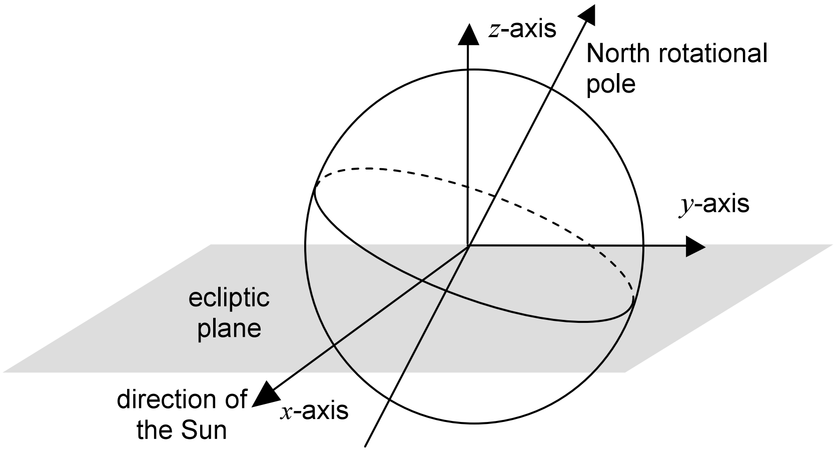
ORM label	Published name
EARTH_INERTIAL_ARIES_1950	Earth equatorial inertial, Aries mean of 1950
EARTH_INERTIAL_ARIES_TRUE_OF_DATE	Earth equatorial inertial, Aries true of date
EARTH_INERTIAL_J2000r0	Earth equatorial inertial, J2000.0
JUPITER_INERTIAL	Jupiter equatorial inertial
MARS_INERTIAL	Mars equatorial inertial
MERCURY_INERTIAL	Mercury equatorial inertial
NEPTUNE_INERTIAL	Neptune equatorial inertial
PLUTO_INERTIAL	Pluto equatorial inertial
SATURN_INERTIAL	Saturn equatorial inertial
URANUS_INERTIAL	Uranus equatorial inertial
VENUS_INERTIAL	Venus equatorial inertial

7.5.3 Solar ecliptic

The *solar ecliptic OBRS* is specified in Table 7.39. See [BHAV, 3.2.5].

Table 7.39 — Solar ecliptic OBRS

Element	Value
OBRS label	SOLAR_ECLIPTIC
OBRS code	2
Short name	solar ecliptic
Object restrictions	A planet in the solar system.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD X_AXIS_3D is in the ecliptic plane and points in the direction of the Sun. 3) The RD Z_AXIS_3D is perpendicular to the ecliptic plane and points northward.

Element	Value
Figures	 NOTE The x-axis slowly rotates once per orbit of the planet around the Sun. The y-axis also lies in the ecliptic plane.
References	[HAPG]

EXAMPLE The *obliquity of the ecliptic* $\varepsilon(t)$ is the angle in radians from the equatorial plane to ecliptic.

The *ecliptic longitude of the Sun* $\lambda_{\square}(t)$ is the angle in radians from the first point of Aries and the line from the centre of the Earth to the centre of the Sun. The direction to the Sun is represented by $\lambda_{\square}(t)$.

The Greenwich sidereal hour angle $\theta_{\mathrm{GSH}}(t)$ is the angle in radians from the first point of Aries to the direction of the x-axis of ORM WGS_1984 Earth reference ORM.

$\theta_{\mathrm{GSH}}(t)$ and $\lambda_{\square}(t)$ depend on the epoch of definition of the first point of Aries. $\theta_{\mathrm{GSH}}(t)$, $\varepsilon(t)$, and $\lambda_{\square}(t)$ are functions of UTC time t elapsed from a given epoch. Approximations of these functions are published in astronomical almanacs and other documents (see [SEID] or [USNOA]).

Given an ERM in the solar ecliptic OBRS, the ERM RT is specified by STT CF_XZ_ROTATE with dynamic parameters:

$$\omega_1(t) = -\varepsilon(t)$$
$$\omega_3(t) = \theta_{\mathrm{GSH}}(t) - \lambda_{\square}(t).$$

Table 7.40 is a directory of the Earth and other planet object solar ecliptic ORMs.

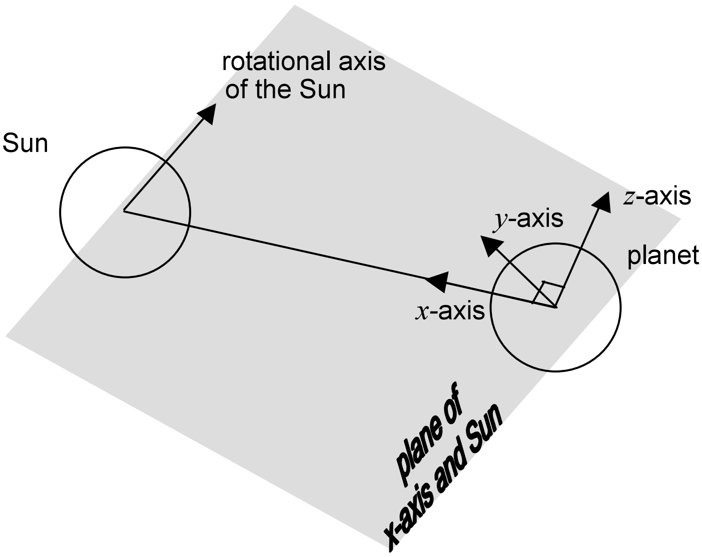
Table 7.40 — Solar ecliptic ORM directory

ORM label	Published name
EARTH_SOLAR_ECLIPTIC	Solar ecliptic
JUPITER_SOLAR_ECLIPTIC	Jupiter solar ecliptic

7.5.4 Solar equatorial

A *solar equatorial OBRS* is specified in Table 7.41.

Table 7.41 — Solar equatorial OBRS

<table>
<tr><th>Element</th><th>Value</th></tr>
<tr><td>OBRS label</td><td>SOLAR_EQUATORIAL</td></tr>
<tr><td>OBRS code</td><td>3</td></tr>
<tr><td>Short name</td><td>solar equatorial</td></tr>
<tr><td>Object restrictions</td><td>A planet in the solar system for which the ecliptic plane is distinct from the equatorial plane.</td></tr>
<tr><td>Object binding rules</td><td>1) The RD ORIGIN_3D is the mass-centre of the planet.
2) The RD X_AXIS_3D is in the ecliptic plane and points in the direction of the Sun.
3) The RD Z_AXIS_3D is perpendicular to the RD X_AXIS_3D in the plane determined by the RD X_AXIS_3D and the rotational axis of the Sun and points northward.</td></tr>
<tr><td>Figures</td><td>

NOTE The xz-plane of a solar equatorial ORM contains the rotational axis of the Sun. The x-axis slowly rotates once per orbit of the object around the Sun. The y-axis is parallel to the solar equatorial plane and points towards dusk [BHAV, 3.2.6].</td></tr>
<tr><td>References</td><td>[CRUS]</td></tr>
</table>

Table 7.42 is a directory of the Earth and other planet solar equatorial ORMs.

Table 7.42 — Solar equatorial ORM directory

ORM label	Published name
EARTH_SOLAR_EQUATORIAL	Solar equatorial
JUPITER_SOLAR_EQUATORIAL	Jupiter solar equatorial

7.5.5 Heliocentric Aries ecliptic

The *heliocentric Aries ecliptic OBRS* is specified for a planet in Table 7.43. See [HAPG].

Table 7.43 — Heliocentric Aries ecliptic OBRS

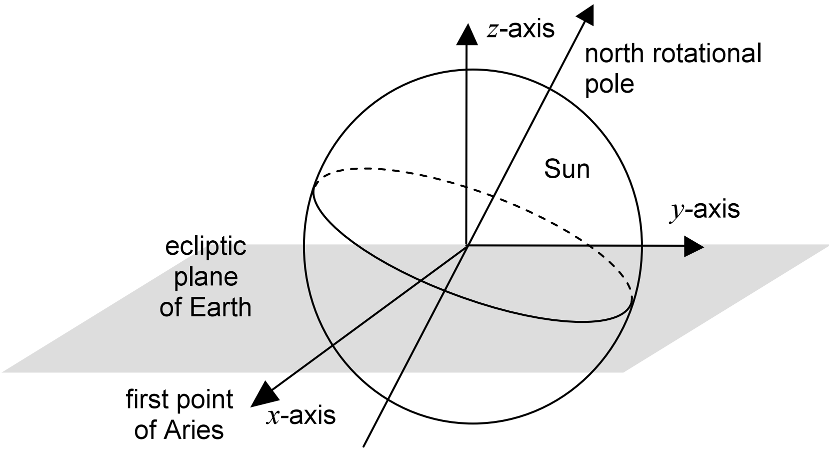
Element	Value
OBRS label	HELIOCENTRIC_ARIES_ECLIPTIC
OBRS code	4
Short name	heliocentric Aries ecliptic
Object restrictions	The Sun.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the Sun. 2) The RD Z_AXIS_3D is perpendicular to the ecliptic plane of the Earth and points in the direction of rotational northwards. 3) The RD X_AXIS_3D is in the ecliptic plane of the Earth and points towards the first point of Aries.
Figures	
References	[HAPG]

Table 7.44 is a directory of heliocentric Aries ecliptic ORMs. The heliocentric Aries ecliptic axis directions are inertial for ORM HELIO_ARIES_ECLIPTIC_J2000r0 and quasi-inertial ORM HELIO_ARIES_ECLIPTIC_TRUE_OF_DATE.

Table 7.44 — Heliocentric Aries ecliptic ORM directory

ORM label	Published name
HELIO_ARIES_ECLIPTIC_J2000r0	Heliocentric Aries ecliptic, J2000.0
HELIO_ARIES_ECLIPTIC_TRUE_OF_DATE	Heliocentric Aries ecliptic, true of date

7.5.6 Heliocentric planet ecliptic

The *heliocentric planet ecliptic OBRS* is specified for a planet in Table 7.45. See [HAPG].

Table 7.45 — Heliocentric planet ecliptic OBRS

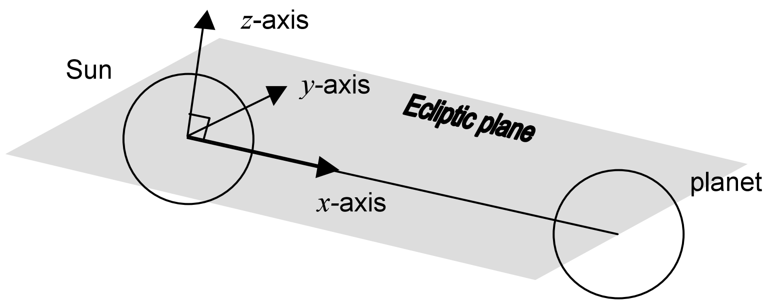
Element	Value
OBRS label	HELIOCENTRIC_PLANET_ECLIPTIC
OBRS code	5
Short name	heliocentric planet ecliptic
Object restrictions	The Sun.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the Sun. 2) The RD Z_AXIS_3D is perpendicular to the ecliptic plane for a specified planet and points in the direction of rotational northwards. 3) The RD X_AXIS_3D is in the ecliptic plane of the specified planet and points towards the planet.
Figures	
References	[HAPG]

Table 7.46 is a directory of heliocentric planet ecliptic ORMs.

Table 7.46 — Heliocentric planet ecliptic ORM directory

ORM label	Published name
HELIO_EARTH_ECLIPTIC	Heliocentric Earth ecliptic

7.5.7 Heliocentric planet equatorial

The *heliocentric planet equatorial OBRS* is specified for a planet in Table 7.47.

Table 7.47 — Heliocentric planet equatorial OBRS

Element	Value
OBRS label	HELIOCENTRIC_PLANET_EQUATORIAL
OBRS code	6
Short name	heliocentric planet equatorial
Object restrictions	The Sun.

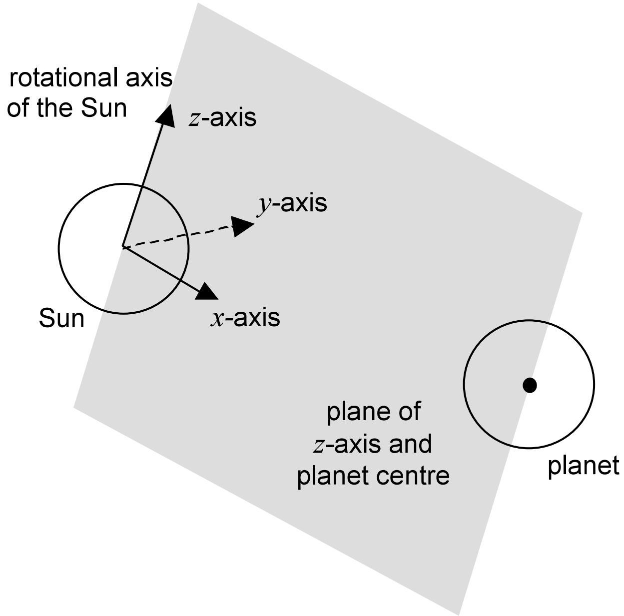
Element	Value
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the Sun. 2) The RD Z_AXIS_3D is perpendicular to the solar equatorial plane of the specified planet and points in the direction of rotational northwards. 3) The RD X_AXIS_3D is perpendicular to the RD Z_AXIS_3D in the plane determined by the RD Z_AXIS_3D and the planet centre and points towards the specified planet.
Figures	
References	[HAPG]

Table 7.48 is a directory of heliocentric planet equatorial ORMs.

Table 7.48 — Heliocentric planet equatorial ORM directory

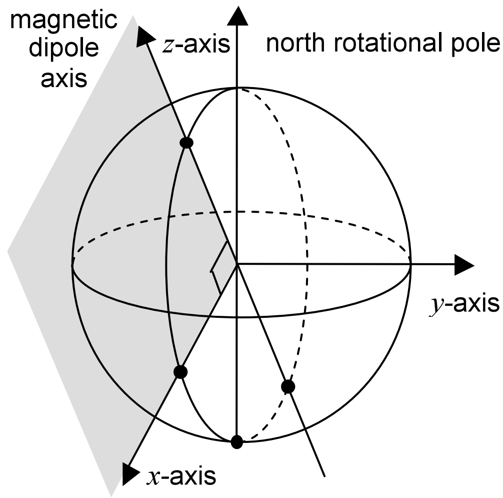
ORM label	Published name
HELIO_EARTH_EQUATORIAL	Heliocentric Earth equatorial

7.5.8 Celestiomagnetic

The *celestiomagnetic OBRS* is specified in Table 7.49. See [BHAV, 3.3.1].

Table 7.49 — Celestiomagnetic OBRS

Element	Value
OBRS label	CELESTIOMAGNETIC
OBRS code	7
Short name	celestiomagnetic
Object restrictions	A planet or rotating satellite in a solar system with a magnetic dipole axis distinct from its rotational axis.

Element	Value
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD Z_AXIS_3D is parallel to the magnetic dipole axis and points towards magnetic north. 3) The RD X_AXIS_3D is contained in the plane through the origin that is parallel to the dipole and rotational axes, perpendicular to the RD Z_AXIS_3D and pointing away from the dipole northward direction.
Figures	 NOTE The rotational south pole is contained in the x-positive xz-half-plane unless the planet has retrograde rotation. This binding is not applicable to Saturn whose magnetic and rotational poles are not distinguished.
References	[CRUS]

In the case of the Earth, this dynamic ERM is approximated as an Earth-fixed ERM for a five-year epoch. The other celestial objects that have observed magnetic dipoles have object-fixed ORM approximations for the corresponding dynamic ORMs.

Table 7.50 is a directory of the Earth and other object-fixed celestiomagnetic ORM approximations.

Table 7.50 — Celestiomagnetic ORM directory

ORM label	Published name
GEOMAGNETIC_1945	DGRF 1945
GEOMAGNETIC_1945_IGRF11	IGRF-11 1945
GEOMAGNETIC_1950	DGRF 1950
GEOMAGNETIC_1950_IGRF11	IGRF-11 1950
GEOMAGNETIC_1955	DGRF 1955
GEOMAGNETIC_1955_IGRF11	IGRF-11 1955
GEOMAGNETIC_1960	DGRF 1960
GEOMAGNETIC_1960_IGRF11	IGRF-11 1960

ORM label	Published name
GEOMAGNETIC_1965	DGRF 1965
GEOMAGNETIC_1965_IGRF11	IGRF-11 1965
GEOMAGNETIC_1970	DGRF 1970
GEOMAGNETIC_1970_IGRF11	IGRF-11 1970
GEOMAGNETIC_1975	DGRF 1975
GEOMAGNETIC_1975_IGRF11	IGRF-11 1975
GEOMAGNETIC_1980	DGRF 1980
GEOMAGNETIC_1980_IGRF11	IGRF-11 1980
GEOMAGNETIC_1985	DGRF 1985
GEOMAGNETIC_1985_IGRF11	IGRF-11 1985
GEOMAGNETIC_1990	DGRF 1990
GEOMAGNETIC_1990_IGRF11	IGRF-11 1990
GEOMAGNETIC_1995	IGRF 1995
GEOMAGNETIC_1995_IGRF11	IGRF-11 1995
GEOMAGNETIC_2000	IGRF 2000
GEOMAGNETIC_2000_IGRF11	IGRF-11 2000
GEOMAGNETIC_2005_IGRF11	IGRF-11 2005
JUPITER_MAGNETIC_1993	Jupiter magnetic
NEPTUNE_MAGNETIC_1993	Neptune magnetic
SATURN_MAGNETIC_1993	Saturn magnetic
URANUS_MAGNETIC_1993	Uranus magnetic

7.5.9 Solar magnetic ecliptic

The *solar magnetic ecliptic* OBRS is specified in Table 7.51. See [BHAV, 3.3.4].

Table 7.51 — Solar magnetic ecliptic OBRS

Element	Value
OBRS label	SOLAR_MAGNETIC_ECLIPTIC
OBRS code	8
Short name	solar magnetic ecliptic
Object restrictions	A planet in the solar system with a magnetic dipole.
Object binding rules	1) The RD ORIGIN_3D is the mass-centre of the planet. 2) The RD X_AXIS_3D is in the ecliptic plane of the planet pointing in the direction of the Sun. 3) The RD Z_AXIS_3D is perpendicular to the RD X_AXIS_3D and points in the direction of rotational northwards in the plane determined by the x-axis and the planetary magnetic dipole axis.

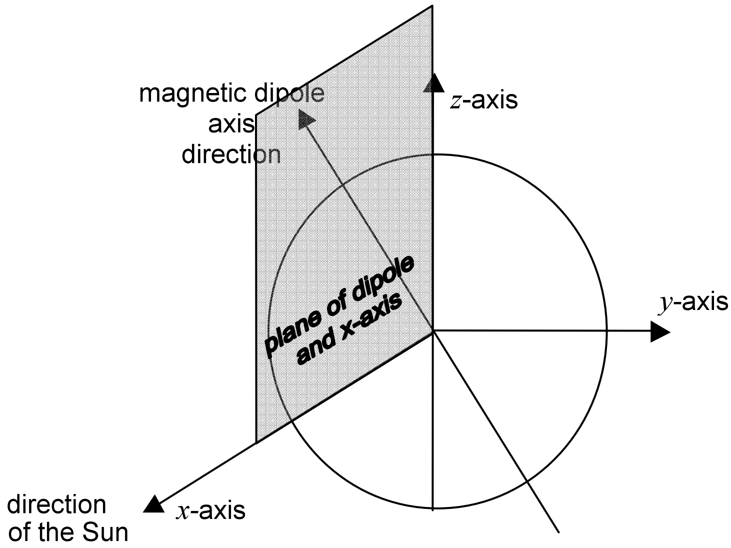
Element	Value
Figures	
References	[CRUS]

Table 7.52 is a directory of the Earth and other planet solar magnetic ecliptic ORMs.

Table 7.52 — Solar magnetic ecliptic ORM directory

ORM label	Published name
EARTH_SOLAR_MAGNETOSPHERIC	Solar magnetospheric
JUPITER_SOLAR_MAG_ECLIPTIC	Jupiter solar magnetic ecliptic

7.5.10 Solar magnetic dipole

The *solar magnetic dipole OBRS* is specified in Table 7.53. See [BHAV, 3.3.5].

Table 7.53 — Solar magnetic dipole OBRS

Element	Value
OBRS label	SOLAR_MAGNETIC_DIPOLE
OBRS code	9
Short name	solar magnetic dipole
Object restrictions	A planet in the solar system with a magnetic dipole.

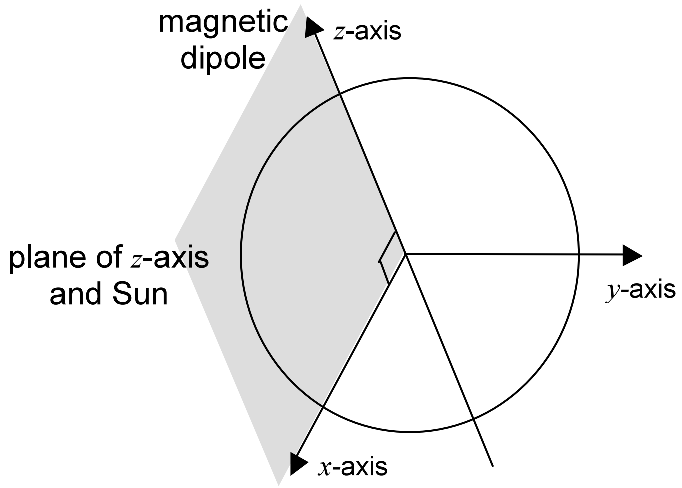
<table>
<tr><th>Element</th><th>Value</th></tr>
<tr><td>Object binding rules</td><td>1) The RD ORIGIN_3D is the mass-centre of the planet.
2) The RD Z_AXIS_3D is parallel to the planetary magnetic dipole axis and points towards magnetic north.
3) The RD X_AXIS_3D is perpendicular to the RD Z_AXIS_3D and pointing towards the Sun in the plane determined by the Sun and the RD Z_AXIS_3D.</td></tr>
<tr><td>Figures</td><td>
</td></tr>
<tr><td>References</td><td>[CRUS], [BHAV]</td></tr>
</table>

Table 7.54 is a directory of the Earth and other celestial object solar magnetic dipole ORMs.

Table 7.54 — Solar magnetic dipole ORM directory

ORM label	Published name
EARTH_SOLAR_MAG_DIPOLE	Solar magnetic dipole
JUPITER_SOLAR_MAG_DIPOLE	Jupiter solar magnetic dipole

